

1. Evaluate  $\log_{2018} \sin x + \log_{2018} \cos x + \log_{2018} \tan x$  where  $x = \pi$ .

- A)  $-1$                   B)  $0$                   C)  $1$                   D) *undefined*          E) *NOTA*

2. Simplify:  $((2^3 + 7^0)^{\frac{3}{2}} + (27^{\frac{1}{3}} - 36^{\frac{1}{2}})^2)^{\frac{1}{2}}$

- A)  $2\sqrt{6}$                   B)  $6$                   C)  $9$                   D)  $\sqrt{6}$                   E) *NOTA*

3. If  $\log_a b + \log_b a = 4$ , what is the value of  $(\log_a b)^2 + (\log_b a)^2$ ?

- A)  $14$                   B)  $15$                   C)  $13$                   D)  $7$                   E) *NOTA*

4. Simplify:  $\frac{1}{\log_7 2} \div \log_{\frac{1}{3} 9} + \log_8 7$

- A)  $-\frac{1}{6} \log_2 7$           B)  $\frac{5}{6} \log_2 7$           C)  $\frac{2}{3}$                   D)  $3$                   E) *NOTA*

5. Find the product of all real solutions to  $x^{\log_{16} x} = 8$ .

- A)  $48$                   B)  $4\sqrt{3}$                   C)  $12$                   D)  $8\sqrt{3}$                   E) *NOTA*

6. Given that  $(\log_x 25)(\log_4 49)(\log_{27} x)(\log_{125} 64)(\log_x 81) = 2$ , find  $\log_7 x$ .

- A)  $\frac{4}{3}$                   B)  $\frac{3}{8}$                   C)  $\frac{8}{3}$                   D)  $\frac{3}{4}$                   E) *NOTA*

7. The value of  $\sqrt{2 + \sqrt{2^2 + \sqrt{2^4 + \sqrt{2^8 + \dots}}}}$ , where exponents are the increasing powers of 2, can be expressed as  $\frac{\sqrt{a} + \sqrt{b}}{2}$  where  $a$  and  $b$  are positive integers. Find  $|a - b|$ .

- A)  $6$                   B)  $8$                   C)  $10$                   D)  $12$                   E) *NOTA*

8. Solve for  $x$ :  $\ln(1 + \log_2(3 + \log_4(5 + x))) = 0$

- A)  $-\frac{79}{16}$                   B)  $-1$                   C)  $-\frac{319}{64}$                   D)  $-\frac{19}{4}$                   E) *NOTA*

9. The roots of  $9x^3 - 28x^2 + 81x - 3 = 0$  are  $r$ ,  $s$ , and  $t$ . Find  $\log_3 r + \log_3 s + \log_3 t$ .

- A)  $3$                   B)  $-2$                   C)  $1$                   D)  $-1$                   E) *NOTA*

10. If  $x = \sqrt[5]{-41}$ , then how many of the following are false?

- I.  $x > -2$       II.  $x^3 \leq -8$       III.  $\sqrt{-x} > 1$       IV.  $x^2 < 4$       V.  $x^4 \geq 16$

- A) 1      B) 2      C) 3      D) 4      E) NOTA

11. Let the roots of the function  $f(x) = x^{20} - 7x^3 + 1$  be  $r_1, r_2, r_3 \dots r_{20}$ . If

$$\frac{a}{b} = \frac{1}{1+r_1^2} + \frac{1}{1+r_2^2} + \frac{1}{1+r_3^2} + \dots + \frac{1}{1+r_{20}^2}$$

where  $a$  and  $b$  are relatively prime positive integers. Find  $a - b$ .

- A) 134      B) 196      C) 81      D) 45      E) NOTA

12. Simplify  $(\log_{2a} 4^x)(1 + \log_2 a)$ .

- A)  $x^2$       B)  $2^x$       C)  $x$       D)  $2x$       E) NOTA

13. Solve for  $x$ :  $3^{x-4} = 4^{x-3}$

- A)  $\frac{\ln 8 - \ln 9}{\ln 3 - \ln 2}$       B)  $\frac{\ln 81 - \ln 64}{\ln 4 - \ln 3}$       C)  $\frac{\ln 64 - \ln 81}{\ln 4 - \ln 3}$       D)  $\frac{\ln 9 - \ln 8}{\ln 3 - \ln 2}$       E) NOTA

14. Let  $F(x) = 2^x + 7$ . The graph of a function  $G(x)$  is produced by expanding the graph of  $F(x)$  vertically by a factor of 8 against the  $x$ -axis and then shifting the resulting graph 8 units up. Which of the following is an equation of  $G(x)$ ?

- A)  $G(x) = 2^{3x} + 15$       C)  $G(x) = 2^{3x} + 64$       E) NOTA  
 B)  $G(x) = 2^{x+3} + 15$       D)  $G(x) = 2^{x+3} + 64$

15. Given  $\log_{n^2} m + \log_{m^2} n = 1$ , solve for  $n$  in terms of  $m$  if  $n \neq 1, m \neq 1$ , and  $m$  is positive.

- A)  $\sqrt{m}$       B)  $m^2$       C)  $m$       D)  $\frac{1}{m}$       E) NOTA

16. Given  $a \log_{1440} 5 + b \log_{1440} 2 + c \log_{1440} 3 = d$ , where  $a, b, c$ , and  $d$  are relatively prime positive integers, find the value of  $ab + cd$ .

- A) 11      B) 9      C) 8      D) 7      E) NOTA

17. Let  $a + \sqrt{b} = \sqrt{6 + (1 + \sqrt{3 + (1 + \sqrt{3 + \sqrt{8}})^2})^2}$  where  $a$  and  $b$  are positive integers.

Find  $a + b$ .

- A) 8                      B) 9                      C) 13                      D) 10                      E) NOTA

18. Given  $\log_x y + \log_y x = \frac{10}{3}$  and  $xy = 400$  where  $x > y > 0$ , find  $x - y$ .

- A)  $8\sqrt{2}$                       B)  $38\sqrt{5}$                       C)  $35\sqrt{2}$                       D)  $16\sqrt{5}$                       E) NOTA

19. Given  $\log_4(\log_{64} x) = \log_{64}(\log_4 x)$ , find the value of  $(\log_2 x)^2$ .

- A) 48                      B) 72                      C) 192                      D) 108                      E) NOTA

20. How many digits are in the decimal form representation of  $18^{50}$ ?

- A) 61                      B) 62                      C) 63                      D) 64                      E) NOTA

21. Find the sum of the digits of the greatest integer less than  $(3 + \sqrt{7})^6$ .

- A) 19                      B) 18                      C) 21                      D) 20                      E) NOTA

22. Evaluate:

$$\begin{vmatrix} 0 & 0 & e^1 \\ 0 & e^3 & e^2 \\ e^1 & e^2 & e^3 \end{vmatrix} + \begin{vmatrix} e^3 & e^2 & e^1 \\ e^2 & e^3 & 0 \\ e^1 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} e^3 & 0 & 0 \\ e^2 & e^3 & 0 \\ e^1 & e^2 & e^3 \end{vmatrix} + \begin{vmatrix} e^3 & e^2 & e^1 \\ 0 & e^3 & e^2 \\ 0 & 0 & e^3 \end{vmatrix}$$

- A) 0                      B)  $\frac{2}{e}$                       C)  $-2e^{-4}$                       D)  $2e^{-4}$                       E) NOTA

23. Find the units digit of  $3^{5^{7^9}}$

- A) 1                      B) 3                      C) 7                      D) 9                      E) NOTA

